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The unabridged, fully illustrated supplement to:

How free am I, really?

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Summary: This supplement presents explanations and illustrations which were omitted from the abridged article [20] to keep it focused on the story of discovery. On the other hand, you may find – as do I – that further meaning within the story is revealed with the companion material presented here. Every illustration was made either with Wolfram's Mathematica [37] or with a picture environment from the LaTeX System [32] used to typeset this manuscript. The contents are as follows.

Contents

1 Pauli's Hexagram Dream

1.1 The Dream

This dream is cited in [20] to introduce the geometric *tetrahedron* [33] and *merkaba* [25] as well as related aspects of the I Ching [36]. The dream appears in [35] (p. 108) as follows:

Dream: A Chinese woman (elevated to the rank of a "Sophia") is present with two men. I am the fourth. She says to me: "You must allow us to play every conceivable combination of chess." In a subsequent half-waking fantasy she announced to the dreamer, in a numinous voice: "In your drawings [Fig. 1] *one element is perfectly correct and another transitory and false. It is correct that the lines number six, but it is false to draw six points. See here–" and I saw* [Fig. 1a] *a square with clearly marked off diagonals. "Can you see now finally the four and the six? Four spatial points and six lines or six pairs out of four points. They are the same six lines that exist in the I Ching. There the six, containing three as a latent factor, are correct. Now observe the square more closely: four of the lines are of equal length, the other two are longer – they are "irrationally related". There is* no *figure with four points and six equal lines.* For this reason symmetry cannot be statically produced and a dance results. *The* coniunctio *refers to the exchange of places during this dance. One can also speak of a game or rhythms and rotations.* Therefore the three, already contained in a latent form in the square, must be dynamically expressed.

(a) (b)

Figure 1: (a) A square shown with its diagonals. (b) A representation of the Star of David using two interpenetrating triangles.

Significant references in the dream to chess and to the I Ching [36] are summarized briefly as follows, and details are given subsequently. The chessboard has 64 squares, an allusion to the number of hexagrams in the I Ching; see the table in Fig. 20 presented later. That all possible games of chess be played seems a hint to the 64×64 possible outcomes from consulting the I Ching; see the table in Fig. 22 presented later. The numerical properties indicated here for the I Ching may be demonstrated as follows.

1.2 Amplifications of I Ching References

To consult the I Ching, one tosses three coins six times. Each coin is labeled heads (H) and tails (T) on opposite sides. The table in Fig. 2 lists the possible results for a single toss. The meaning of the table entries is explained in the caption. For six tosses, there are six such results, each having a present and a future state. These states are represented as yin (broken) or yang (unbroken) lines. The six present states form a so-called hexagram for the present, and the six future states form a hexagram for the future. Two illustrative examples are given in Fig. 3.

In the example of Fig. 3a, tosses 2, 4 and 6 give 3T, and tosses 1, 3 and 5 give 2T,1H. Following the table in Fig. 2, the resulting lines are old yin and young yang, respectively. Before changing the old yin lines, the present hexagram is given in the third column. This alternating pattern of yin and yang is hexagram 63, which is found in [36] with the name *After Completion*. After changing lines are changed,

coin toss	lines	behavior	name	probability
3H	⊖	yang ⚊ in the present, yin ⚋ in the future	old yang	1/8
2H, 1T	⚋	yin ⚋ in both present and future	young yin	3/8
2T, 1H	⚊	yang ⚊ in both present and future	young yang	3/8
3T	⊗	yin ⚋ in the present, yang ⚊ in the future	old yin	1/8

Figure 2: Ignoring the order of H and T, there are 4 possible results of tossing 3 coins, each labeled H and T on opposite sides. These are shown in column 1. The I Ching assigns to these results the respective lines shown in column 2. Each line has a present state and a future state, shown in column 3, which can be either yin ⚋ or yang ⚊. If the line does not change from present to future, it is called young; otherwise it is called old. The corresponding names are shown in column 4. Under the assumption that H and T are equally probable, the probabilities of the respective results are shown in column 5.

	coin toss	lines	present	future
toss 6:	3T	⊗	⚋	⚊
5:	2T,1H	⚊	⚊	⚊
4:	3T	⊗	⚋	⚊
3:	2T,1H	⚊	⚊	⚊
2:	3T	⊗	⚋	⚊
toss 1:	2T,1H	⚊	⚊	⚊

(a)

coin toss	lines	present	future
2H,1T	⚋	⚋	⚋
3H	⊖	⚊	⚋
2H,1T	⚋	⚋	⚋
3H	⊖	⚊	⚋
2H,1T	⚋	⚋	⚋
3H	⊖	⚊	⚋

(b)

Figure 3: Two examples of consulting the I Ching leading in both cases to the present hexagram 63 *After Completion*. (a) This case leads to the future hexagram 1, *The Creative*. (b) This case leads to the future hexagram 2, *The Receptive*.

the future hexagram is given in the fourth column. This 6-fold of yang is hexagram 1 with the name *The Creative* [36].

In the example of Fig. 3b, tosses 1, 3 and 5 give 3H, and tosses 2, 4 and 6 give 2H,1T. Following the table in Fig. 2, the resulting lines are old yang and young yin, respectively. Before changing the old yang lines, the present hexagram is given in the third column. This is the same alternating pattern of yin and yang as seen in Fig. 3a, i.e., hexagram 63 with the name *After Completion* [36]. After changing lines are changed, the future hexagram is given in the fourth column. This 6-fold of yin is hexagram 2 with the name *The Receptive* [36].

Note that in any position of a hexagram, there are exactly 2 possible states, yin or yang. Since there are 6 separate positions in the hexagram, there are $2^6 = 64$ possible hexagrams. This is the cited number of hexagrams [36], and Fig. 20 shows my ordering of them.

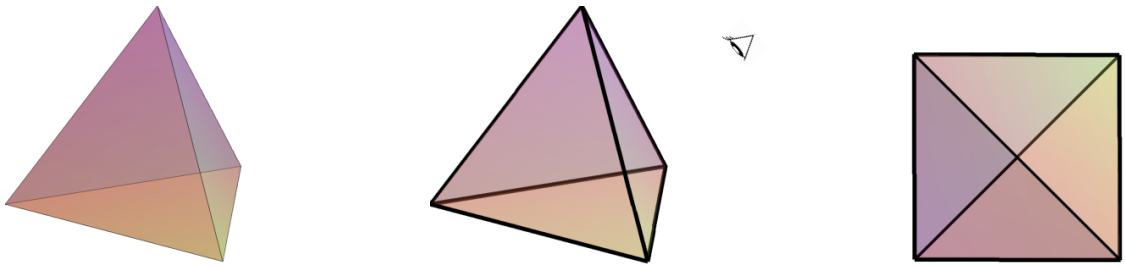
In the examples of Figs. 3a and 3b, hexagram 63 is the present hexagram in both cases, but after changing lines are changed, hexagram 1 results in Fig. 3a and hexagram 2 results in Fig. 3b. Furthermore, one can choose any arbitrary target future hexagram in the fourth column and work backwards from hexagram 63 in the third column to create lines in column 2 and coin tosses in column 1 which will correspond to the chosen future hexagram. So any of the 64 hexagrams could have been obtained for the future after starting in the present with hexagram 63. In fact, any of the 64 hexagrams can be obtained for the future after starting in the present with any of the 64 hexagrams. This means that there are 64×64 possible results from consulting the I Ching.

Actually, this result can be seen directly from Fig. 2. There are exactly 4 possible states, old and young yin and yang, for 6 separate coin tosses; so there are $4^6 = 64^2$ possible pairs of present and future hexagrams.

1.3 Amplifications of the Dream Figures

The Chinese woman claims that there is no figure with four points and six equal lines. In the footnote in [35] on p. 109, M.-L. von Franz amplifies the claim by pointing out: "The statement that there is no figure with four points and six equal lines is only valid for two-dimensional structures." I then realized that Pauli's 2D form in Fig. 1a is a flat projection of a 3D *tetrahedron* [33], as seen in Fig. 4. Then I realized that his 2D form in Fig. 1b is a flat projection of a 3D *merkaba* [25], as seen in Fig. 5. These forms and relations between them are considered in the following.

A regular tetrahedron [33] is a 3D solid whose boundary is given by three equilateral triangles joined along their edges. The form is rendered with the partially transparent shape shown in Fig. 4a. Its edges are emphasized with the outline in Fig. 4b. From the perspective of the viewing eye in the middle, one sees the square object shown in Fig. 4c. The outline of this square object is identical with that of Pauli's 2D form in Fig. 1a. Thus, Fig. 4 shows the relationship between Pauli's hexagram in Fig. 1a and the tetrahedron shown in Fig. 4a. Note that B. Fuller [5] saw the tetrahedron as the basic building block of the universe.



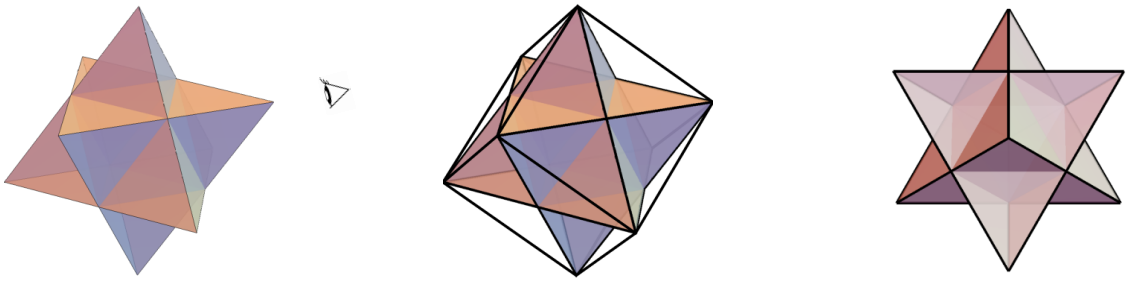
(a) (b) (c)

Figure 4: (a) A regular tetrahedron rendered in 3D. (b) An outline of the tetrahedron emphasizing its edges. The viewing eye gives: (c) A projection of the tetrahedron through opposite perpendicular edges, as illustrated also later in Fig. 11b.

Superimposing an upright tetrahedron with an inverted one gives the 3D solid rendered in Fig. 5a. (As explained in [25], the 3D solid is actually called the *Stella Octangula*, and its wireframe version – the edge set – is called the merkaba. Nevertheless, for ease of exposition, the distinction between the two terms will be blurred, and only the term *merkaba* will be used for the 3D object.) As illustrated in Fig. 5b, the merkaba is given by two overlapping tetrahedra which interpenetrate in such a way that the tetrahedral vertices coincide with the vertices of their circumscribing cube. The name *Merkaba* is Hebrew for *throne chariot*, which refers to the throne chariot of Ezekiel's Vision [3]. This historical aspect of the merkaba is discussed later in the context of representations of the Self.

Figure 5b also shows that the edges of each of the two tetrahedra lie perpendicular to each other on the sides of the circumscribing cube. Their points of intersection lie at the midpoints of the cube faces. From the perspective of the viewing eye shown at the left, one sees the star-shaped object shown in Fig. 5c. The outer boundary of this star-shaped object is identical with Pauli's form shown in Fig. 1b. Thus, Fig. 5 shows the relationship between Pauli's Star of David in Fig. 1b and the merkaba illustrated in Fig. 5a. Note that B. Fuller [5] saw the merkaba as a stable realization of the cube, which would otherwise be structurally unstable.

The richness of these symbols is elaborated in the following sections. Thus, as indicated in [20], the dream symbols gain meaning when boosted to a higher dimension.



(a) (b) (c)

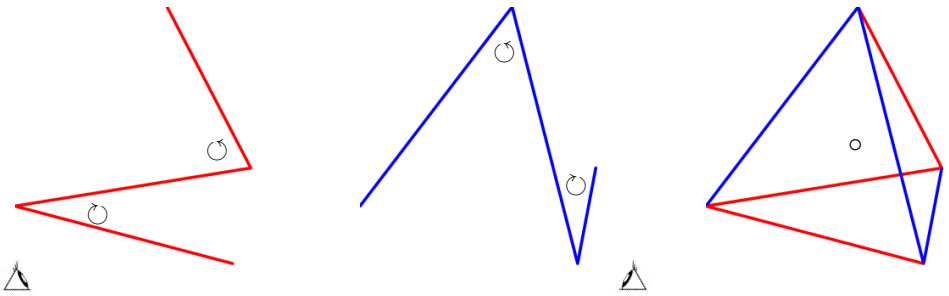
Figure 5: (a) A merkaba rendered in 3D. (b) The two interpenetrating tetrahedra are situated so that their vertices coincide with that of the cube circumscribing the merkaba. (c) The star-shaped collection of edges as seen from the viewing eye shown at the left.

2 Tetrahedral Representation of Hexagrams

As indicated in [2], M.-L. von Franz amplified the square form of Fig. 1a by developing a representation scheme for the hexagrams in which the bars $1-6$ of a hexagram, running from bottom to top as usual, are numbered along the sides and diagonals of the square according to: 1:left, 2:right, 3:bottom, 4:top, 5:slant-up, 6:slant-down. This so-called box representation is used by S.H. Cullinane to create some beautiful geometric illustrations of the hexagrams [2]. I want to present an alternative but related and meaningful representation of the hexagrams based on the tetrahedron.

2.1 Trigrams and Dual Spiral Edge Chains

Figure 6 illustrates that the tetrahedron is formed by linking two spiral chains, each consisting of three edges [5]. This duality suggests the designation of the chains in Fig. 6a and 6b as Yin and Yang chains, respectively. The naming is based on their association with trigrams, which is explained in more detail in the next subsection. The chains are also referred to as broken triangles, since an end-edge can be pivoted to join both end-vertices and thereby form a triangle.



(a) (b) (c)

Figure 6: (a) The yin-chain of edges and (b) the yang-chain of edges combine to form a duality in (c) the tetrahedron. The circular arrows indicated turning directions when the observed point moves from right to left according to perspective of the viewing eye outside the tetrahedron.

When viewed from outside the tetrahedron, the spiral chains of Fig. 6 can be seen as discretizations of the corresponding double spirals of Fig. 7. Specifically, let an observed point move through a given chain from one end to the other. When the chain in Fig. 6a is observed from the position of its viewing eye, the observed point moves first counterclockwise and then clockwise, as in the double spiral of Fig. 7a.

When the chain in Fig. 6b is observed from the position of its viewing eye, the observed point moves in the opposite direction, first clockwise and then counterclockwise, as in the double spiral of Fig. 7b.

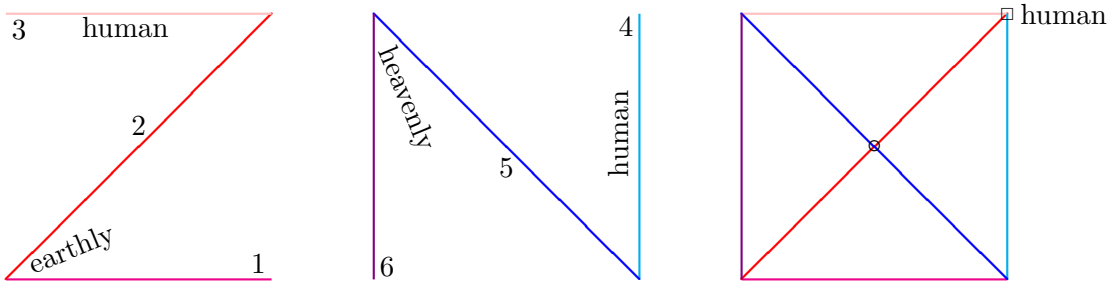
(a) (b)

Figure 7: Celtic double spirals, spiraling (a) initially counterclockwise and finally clockwise and (b) vice-versa.

The bars 1 – 3 in the hexagram are called the lower trigram and bars 4 – 6 are called the upper trigram. The yin- and yang-chains will be associated naturally with the lower and upper trigrams, respectively. As explained in [36], yin is considered feminine and earthly, while yang is considered masculine and heavenly. Furthermore, the lower two bars of a hexagram are considered earthly, the middle two bars human and the top two bars heavenly. The bars 2 – 4 and 3 – 5 are also called nuclear trigrams.

2.2 Mapping the Hexagram to the Tetrahedron

The bars of a hexagram are mapped onto the edges of a tetrahedron as follows. The lower trigram is mapped onto the yin-chain, and the upper trigram is mapped onto the yang-chain. The representation leans on the projection from 3D into 2D as seen in Fig. 4. The pictograms (a), (b) and (c) shown in Fig. 8 correspond to their counterparts in Fig. 6. The lower trigram is illustrated in Fig. 8a with the bars numbered 1 – 3. The upper trigram is illustrated in Fig. 8b with the bars numbered 4 – 6. As mentioned earlier, and illustrated in Fig. 9c, the first two bars of a hexagram are considered earthly, the middle two bars human and the last two bars heavenly. The pictograms of Fig. 8 manifest this association through their correspondence with the principles of image analysis [9] or of sandplay analysis [30], seen as follows.



(a) (b) (c)

Figure 8: The tetrahedral edges have been projected from 3D into 2D as seen in Fig. 4. (a) The edges of the yin-chain are numbered 1 – 3 and correspond to the lower trigram. (b) The edges of the yang-chain are numbered 4 – 6 and correspond to the upper trigram. (c) The edges of the projected tetrahedron correspond to the hexagram. The human vertex connects the two human bars and the two trigrams.

As seen in Fig. 8a, the yin-chain begins lower right, the material corner. The earthly bars channel energy first to the lower left, the corner of the collective unconscious, and then to the upper right, the collective consciousness corner. The human bar channels energy from the collective consciousness to the upper left, the spiritual corner. The first two bars of the chain point conspicuously downward, consistent with the name *earthly*.

As seen in Fig. 8b, the yang-chain begins upper right, the collective consciousness corner. The human bar channels energy from the collective consciousness to the lower right, the material corner. Then the heavenly bars channel energy to the upper left, the spiritual corner, and then to the lower left, the corner of the collective unconscious. The last two bars of the chain point conspicuously upward, consistent with the name *heavenly*.

The yin- and yang-chains are shown together in Fig. 8c, where the human vertex is emphasized at the upper right corner. Note that the human vertex fittingly channels energy from the collective consciousness corner to the material and spiritual corners. Note further that the nuclear trigrams both point to the human vertex.

As an example of this approach, consider the hexagram shown in Fig. 9. It is number 63, *After Completion* [36], considered in Fig. 3. Note that, fittingly, 63 is Jung's grave number. The full tetrahedral representation is shown in Fig. 9a, and its projection according to Fig. 4 is shown in Fig. 9b. The traditional representation is shown in Fig. 9c.

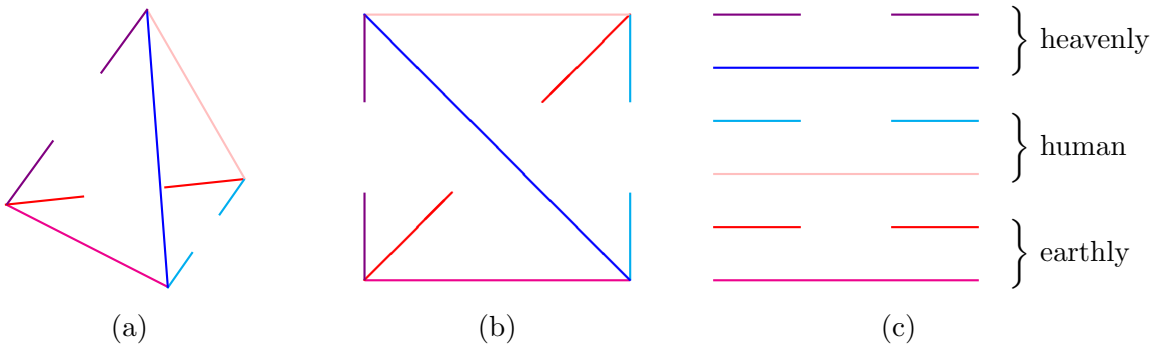


Figure 9: (a) The tetrahedral representation of hexagram 63 and (b) its 2D projection according to Fig. 4. (c) The traditional representation of 63, as considered in Fig. 3. The color-coding, as well as the full or broken lines, are the same among the three representations.

3 Tetrahedral Representation of Psychic Functions

The geometric symbols presented above will be associated here with psychic functions identified by Jung. This is not the first time that such connections have been recognized. For instance, J. Gaboury has elaborated very interesting associations of a related nature [6]. In particular, Gaboury drew my attention to important connections with the work of B. Fuller [5], as mentioned above in relation to the tetrahedron and the merkaba. Also, S.G. Jung has developed a very impressive and detailed geometric treatment of his *reimagined* I Ching in [17] and [18]. Nevertheless, my presentation leans more strongly on Pauli's dreams and on the traditional I Ching.

3.1 Functions in Consciousness

In particular, the attitudes and functions in consciousness can be mapped into the tetrahedron. The attitudes and functions [13] are depicted in the usual criss-cross way in Fig. 10, including the differentiations into the astrological signs associated with the respective elements [24].

In the same way that the symbols of Fig. 1 gain meaning when boosted to a higher dimension, I find that a new perspective on the psychic functions emerges when the flat 2D diagram of Fig. 10 is given a 3D representation. For this, I first summarize certain further geometric properties of the tetrahedron.

Figure 11 shows that the tetrahedron has seven axes of symmetry. An axis of symmetry running through a vertex and perpendicularly through the midpoint of its opposite face is shown in Fig. 11a. There are

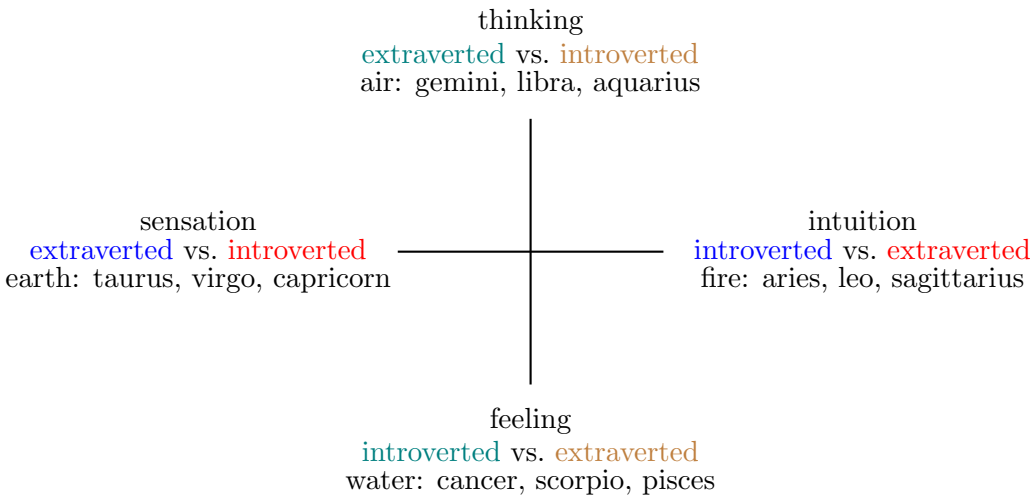


Figure 10: The attitudes and functions in consciousness are shown together with their differentiations into the astrological signs associated with the respective elements. The color-coding is intended to show the polar pairings, e.g., extraverted thinking with introverted feeling.

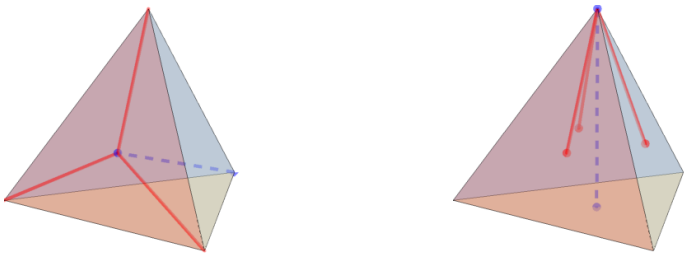
four such axes, one for each vertex. Each is an axis of symmetry because after rotating 120° around the axis, an identically positioned tetrahedron is obtained. An axis of symmetry running through the midpoints of opposite perpendicular edges is shown in Fig. 11b. There are three such axes, one for each pair of opposite edges. Each is an axis of symmetry because after rotating 180° around the axis, an identically positioned tetrahedron is obtained.

(a) (b)

Figure 11: (a) An axis of symmetry running through a vertex and through the midpoint of its opposite face. There are four such axes. (b) An axis of symmetry running through the midpoints of opposite edges. There are three such axes.

Figure 12a shows that a midpoint of a triangular face of the tetrahedron has three spokes, each running from the midpoint to a vertex of the triangle. Since there are four triangular faces, there are four such systems of spokes. So there are 12 facial spokes. Similarly, Fig. 12b shows that a vertex of the tetrahedron has three spokes, each running to a midpoint of a neighboring triangular face. Since there are four tetrahedral vertices, there are four such systems of spokes. So there are 12 verticial spokes.

As discussed beginning at p. 169 of [26], Pauli struggled with the number 12 in relation to Fig. 1, and he even suspected that the zodiac may be incomplete. Yet, the systems of 12 spokes illustrated in Fig. 12, and applied below for the psychic functions, seem to provide a satisfactory basis for the number 12. Furthermore, as discussed later, it is a significant property of the merkaba to have 12 edges, onto which the present and future hexagrams are mapped.



(a) (b)

Figure 12: (a) There are three spokes which run from the midpoint of a triangular face to a vertex. (b) There are three spokes which run from a vertex to the centers of the neighboring faces.

Numbers are discussed with enthusiasm in [26] especially in relation to the *2000-year-old problem*, "How to get from three to four?" Both Jung and Pauli were fascinated and preoccupied with this question. Pauli considered Fig. 1a as a kind of "solution" to the problem, since his hexagram dream calls *three* a latent factor among *four* points, i.e., there are $6 = 3 \times 2$ edges connecting 4 points. I find this explanation vague and limited by a continued restriction to 2D. A much clearer resolution is given by a glance at the tetrahedron in 3D, which unifies the forms of Fig. 1 as well as the numbers 3 and 4. For instance, there are 3 vertices in each of 4 faces of the tetrahedron. Also, 3 and 4 are linked through their respective types of symmetry in Fig. 11, giving a total of 7 axes of symmetry.

The emergence of the alchemical number *seven* is especially unexpected for me, because other numbers are so much more conspicuous: *one* center, *two* spiral edge chains, *three* vertices per face, *four* faces, *five* points among 4 vertices and 1 center, and *six* edges. With an eye toward the I Ching, I associate the six edges with the number of bars in a hexagram; as well, the number $2^6 = 64$ of binary (yin-yang) labelings of the edges with the number of hexagrams. I associate the seven axes of symmetry with the alchemical planets because of their rotational character. In particular, I associate the three axes of symmetry in Fig. 11a with the outer planets: Mars, Jupiter and Saturn. I associate the four axes of symmetry in Fig. 11b with the inner planets: Sun, Mercury, Venus and Moon.

These alchemical and astrological references lead back to the discussion of the psychic functions illustrated in Fig. 10. As planned, that flat diagram will now be given a 3D representation, seen in Figs. 13 and 14.

Because each *vertex* of the tetrahedron appears *outwardly* directed in relation to its opposite face, I map each extraverted function of Fig. 10 into a vertex of the tetrahedron. Then I map the polar introverted function of Fig. 10 into the face opposite that vertex. Finally, the functions are differentiated further along the tetrahedral faces by mapping the corresponding astrological signs into the radiating facial spokes defined in Fig. 12a.

The four colors of Fig. 10 indicate the four possibilities for such a mapping into the tetrahedron. Two of these are illustrated in Fig. 13: extraverted thinking with introverted feeling and extraverted intuition with introverted sensation. As a consequence of such a mapping, each extraverted function on a tetrahedral vertex receives a blend of astrological spokes on adjacent faces, as illustrated in Fig. 12b. In particular, extraverted thinking on the vertex in Fig. 13a is a convergence of astrological spokes from all three elements other than water; further, extraverted intuition on the vertex in Fig. 13b is a convergence of astrological spokes from all three elements other than earth.

3.2 Counterparts in the Unconscious

The constructions of the previous subsection provide a 3D geometric representation for the psychic functions in consciousness. Since Jung suggests that consciousness emerges from its preconfiguration in

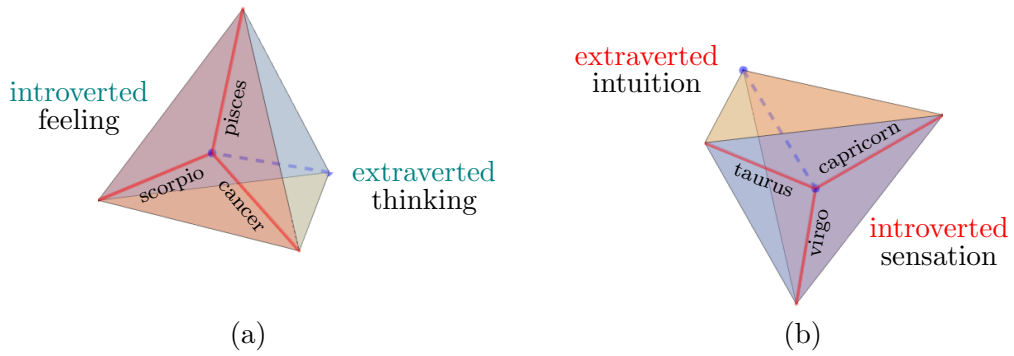


Figure 13: Two examples of mapping entries in Fig. 10 into the tetrahedron are given. (a) A vertical axis in Fig. 10 is illustrated. Extraverted thinking at the vertex is paired with introverted feeling on the opposite face, which differentiates into the water signs. (b) A horizontal axis in Fig. 10 is illustrated. Extraverted intuition at the vertex is paired with introverted sensation on the opposite face, which differentiates into the earth signs.

the unconscious [15], it is natural to seek an analogous 3D representation for psychic functions in the unconscious. I propose the counterpart given in Fig. 14.

As displayed in Fig. 10, the functions in consciousness, thinking, feeling, sensation and intuition, have deep precursors given by air, water, earth and fire, respectively. In the setting of the unconscious, the *outward* direction points from the personal to the collective unconscious. So the counterpart to extraversion in the unconscious is to be directed toward the collective unconscious. The counterpart to introversion in the unconscious is to be directed toward the personal unconscious. Thus, I map each collective function into a vertex of the tetrahedron. Then I map the polar personal function into the face opposite that vertex. Finally, the functions are differentiated further along the tetrahedral faces by mapping the corresponding astrological signs into the radiating facial spokes defined in Fig. 12a.

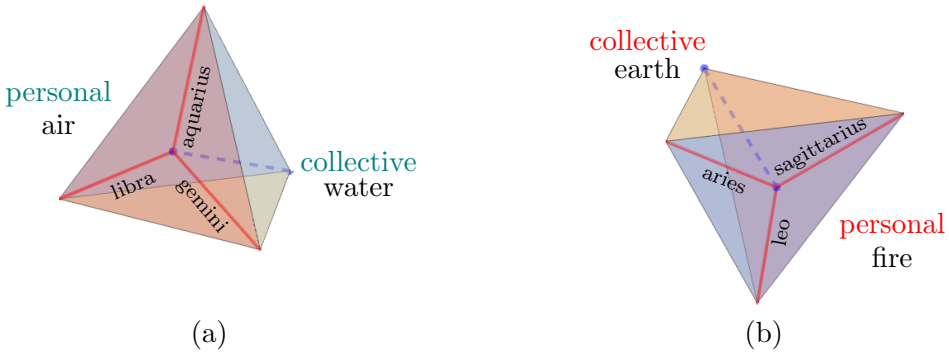


Figure 14: Two examples of mapping functions in the unconscious into the tetrahedron are given. These are counterparts to mapping functions in consciousness as shown in Fig. 13. (a) Collective water at the vertex is paired with personal air on the opposite face, which differentiates into the air signs. (b) Collective earth at the vertex is paired with personal fire on the opposite face, which differentiates into the fire signs.

The color-scheme of Figs. 10 and 13 is used analogously here to map functions in the unconscious into the tetrahedron. Two of these are illustrated in Fig. 14: collective water with personal air and collective earth with personal fire. As a consequence of this mapping, each collective function on a tetrahedral vertex is seen to receive a blend of astrological spokes on adjacent faces, as illustrated in Fig. 12b. In particular, collective water on the vertex in Fig. 14a is a convergence of astrological spokes from the three elements other than air; further, collective earth on the vertex in Fig. 14b is a convergence of astrological spokes from the three elements other than fire.

4 Merkaba Representations of the Self

4.1 Based on Psychic Functions

Tetrahedral representations illustrated in Figs. 13 and 14 for functions in consciousness and in the unconscious, respectively, are now joined to form a merkaba as illustrated in Fig. 15. A model emerges, as discussed in [20], with the Self focused in the center, corresponding to Jung's Liverpool dream [12].

In this geometric union of functions, each tetrahedral face is associated either with an introverted function in consciousness or with a personal function in the unconscious. Each tetrahedral vertex is associated either with an extraverted function in consciousness or with a collective function in the unconscious. The pairing of a face with the vertex extending through it corresponds to the pairings shown in Fig. 10, i.e., sensation with earth, thinking with air, feeling with water and intuition with fire [24]. Building directly on Figs. 13 and 14, two such pairings are illustrated explicitly in Fig. 15a and in Fig. 15b. There are altogether eight such pairings, one near each of the eight vertices.

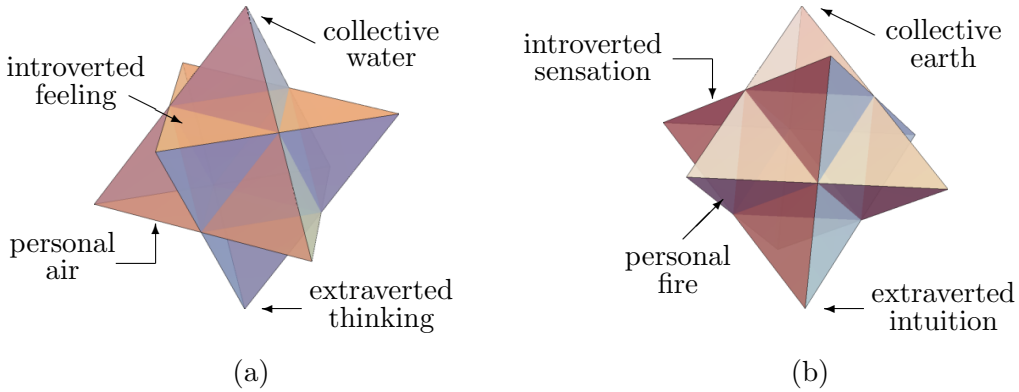


Figure 15: Tetrahedral representations for the functions in consciousness and in the unconscious are shown joined in a merkaba. Two examples of the eight mappings are illustrated explicitly. (a) The vertex associated with collective water extends through the face associated with introverted feeling. The vertex associated associated with extraverted thinking extends through the face associated with personal air. (b) The vertex associated with extraverted intuition extends through the face associated with personal fire. The vertex associated with collective earth extends through the face associated with introverted sensation.

Further geometric and numerical properties of the merkaba are now summarized for what follows. The merkaba circumscribes a regular octahedron [28], shown below in Fig. 17d, whose vertices lie at the midpoints of the cube faces in Fig. 5b. Each group of four coplanar vertices of the octahedron forms a square; cf. the quaternia in Fig. 16. In each case, the two octahedral vertices outside the square serve respectively as north and south poles along the axis perpendicular to the square.

So the circumscribed octahedron contains *three* mutually perpendicular squares, each having *four* vertices. Further, the octahedron has *six* vertices, *eight* faces and *twelve* edges. The *one* center of the octahedron coincides with that of the merkaba and its *two* interpenetrating tetrahedra. There are *eight* small tetrahedra. Each tetrahedron, large or small, has *three* vertices, *four* faces and *six* edges. The total number of large edges is *twelve*, and these edges have a total of $2^{12} = 64^2$ binary (yin-yang) labelings, corresponding to the number of possible results from consulting the I Ching.

4.2 Based on Ezekiel's Vision

The model of Fig. 15 will now be related to Jung's model of the Self [14], which is illustrated in Fig. 16. This model is based largely upon Ezekiel's Vision; see the thorough explanations and especially the artistic illustrations at [3]. Figure 16a shows the sequence of quaternia Jung formulated based upon

correspondences between the psyche and the history of mankind over the last astrological era, roughly 2000 years. He recognized a similar essence shared by the purely spiritual vertex at the top of the sequence and the purely energetic vertex at the bottom of the sequence. So he joined the top and the bottom of this sequence to form the cyclical algebraic representation in Fig. 16b.

The Anthropos, Shadow, Paradise and Lapis quaternia in Fig. 16a correspond, respectively, to the geometric quaternia $a_i, b_i, c_i, d_i,\ i = 0, 1, 2, 3,$ in Fig. 16b. The verticial stages Anthropos, Human, Serpent, Lapis, and Rotundum↔Anthropos correspond, respectively, to the geometric bridges A, B, C, D in Fig. 16b. The Diabolos-Christos dividing line separates the polar opposites, light and dark, conscious and unconscious. These constructs will now be related explicitly with Ezekiel's Vision [3], summarized as follows.

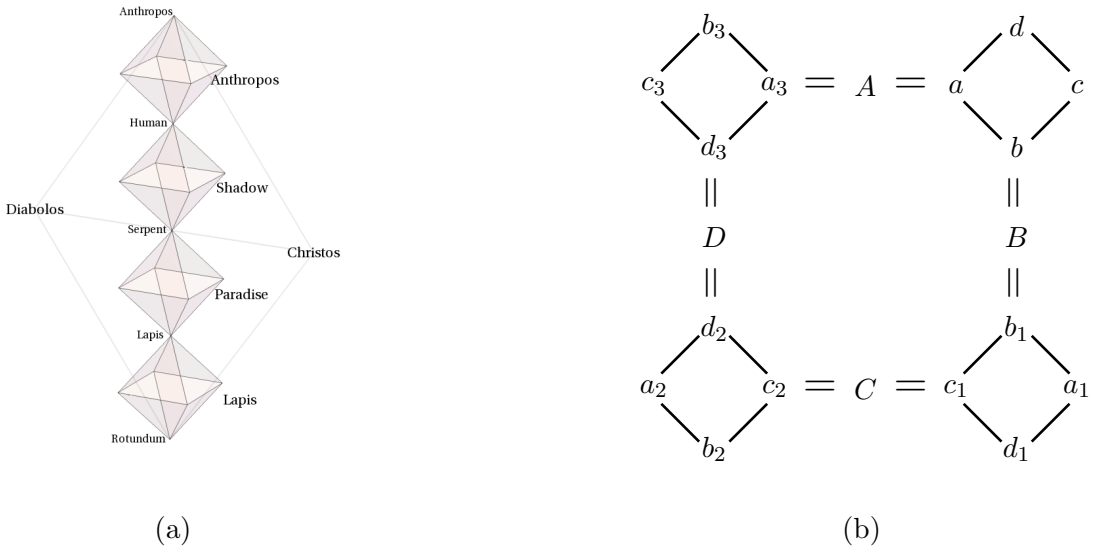


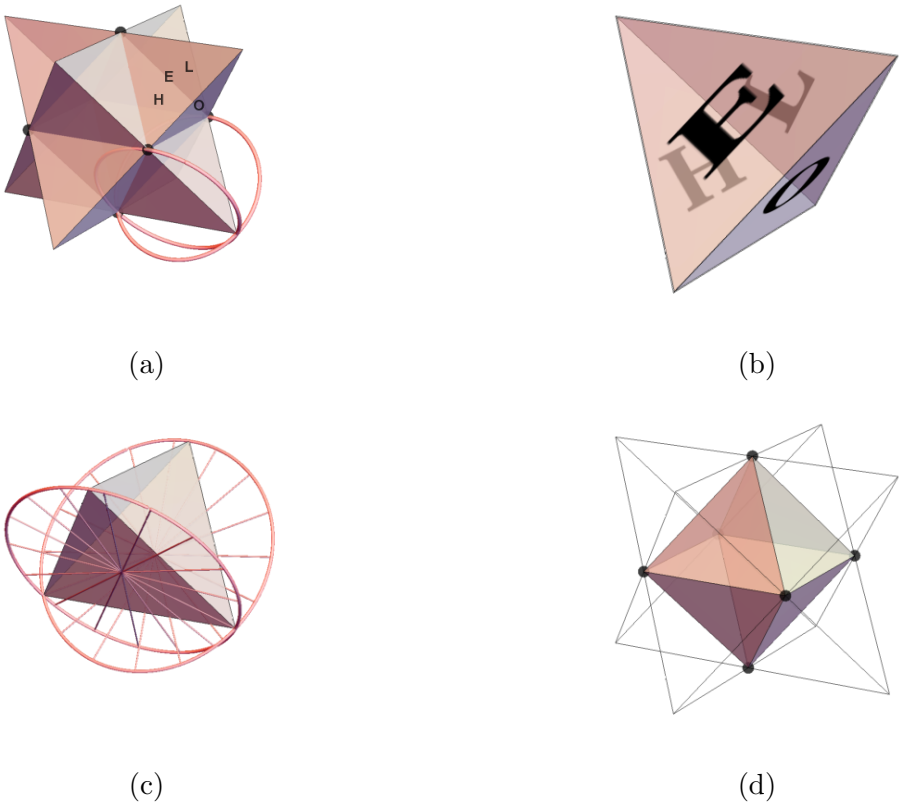
Figure 16: (a) Jung's geometric model of the Self, joining vertices of octahedra. (b) Jung's algebraic formula for the Self.

In his vision, Ezekiel saw four creatures, each with four faces, one human, one of a lion, one of an ox and one of an eagle. He also saw four systems of wheels, each with two independently rotating wheels. Above the creatures, he saw a sapphire stone, like a throne, and upon it he saw the likeness of a man, of God. The entire structure is called the throne-chariot, which is called the *Merkaba* in Hebrew, as indicated earlier. With this association, there is a remarkable correspondence between Ezekiel's Vision and the geometric form of the merkaba, as illustrated in Fig. 17.

Each creature may be identified with one of the small tetrahedra on the top of the merkaba in Fig. 17a. Specifically, each of these tetrahedra has four faces, as does each creature; see the markings E, L, O and H in Fig. 17a. The one small tetrahedron so marked in Fig. 17a has more clearly illustrated faces in Fig. 17b. The human face is on the face not visible from outside the merkaba, and it faces toward the center of the merkaba. The other faces are visible from outside the small tetrahedron.

Each wheel system may be identified with one of the small tetrahedra on the bottom of the merkaba as seen in Fig. 17a. Specifically, each of these tetrahedra has four faces, and each of these faces has a circumscribing circle. Yet only two of these four circles reach the surface on which the merkaba is said to move. These two circles form one wheel system. The one small tetrahedron in Fig. 17a showing the two circles has more clearly illustrated wheels in Fig. 17c.

The sapphire of Ezekiel's vision may be identified with the octahedron in Fig. 17c. With the Self or the God-image identified with the center of the merkaba, Ezekiel's apparent man, indeed God, may be



(a) (b)

(c) (d)

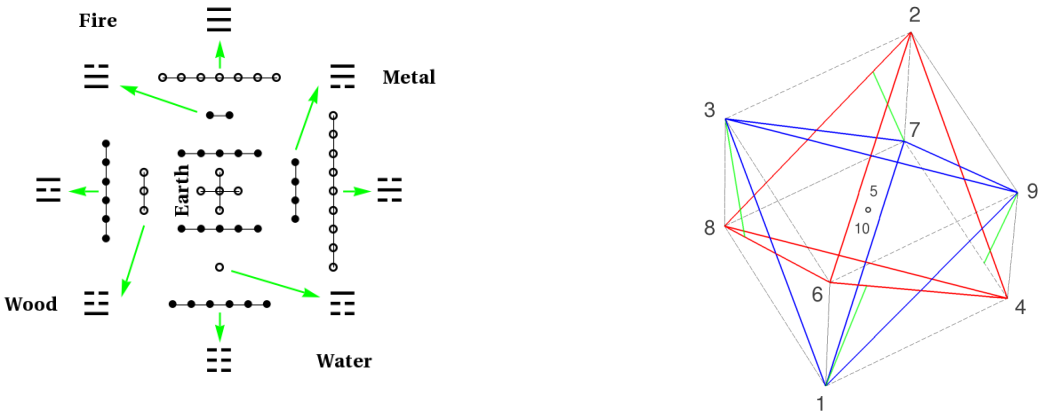
Figure 17: (a) The merkaba with 8 small tetrahedra, 4 on the top and 4 on the bottom. Those on the top represent 4 creature heads, and those on the bottom represent 4 systems of paired wheels. Only one creature head (E,L,O,H) and one system of paired wheels (2 circles) are illustrated, the former over the latter. (b) One such creature head is illustrated with 4 faces, lion (L), ox (O), eagle (E) and human (H), where the human face is directed inward. (c) One of the paired wheel systems. There is one circle defined by the vertices of each face, giving four circles, but only two touch the ground on which the merkaba moves. (d) The octahedron embedded in the merkaba represents the sapphire. Its 6 vertices are highlighted with • in the octahedron and in the merkaba.

situated at the center of the sapphire. Thus, the inwardly directed human face of Fig. 17b is looking to the heavens in the center.

4.3 Jung's Chain of Quaternia

Here I want to combine Jung's model of the Self with other models of this section to establish an integrated whole. Note in [14] that none of Jung's quaternia, Anthropos, Shadow, Paradise or Lapis, are constructed with *polar* elements at opposite corners. So the quaternia can as well, and perhaps more efficiently, be represented in terms of tetrahedra instead of with octahedra as seen in Fig. 16a. For instance, Jung's octahedra in Fig. 16a are connected through the extra vertices outside the squares; alternatively, the small tetrahedra in Fig. 15 are connected by shared edges. So the developmental stages, Anthropos, Human, Serpent, Lapis, and Rotundum↔Anthropos, can as well be represented by the shared edges which form a cycle.

Figure 15 shows that each small tetrahedron from one large tetrahedron sits on a face of the other large tetrahedron. So if a selected small tetrahedron belongs to one system of functions (in consciousness or in the unconscious), then the other small tetrahedra sharing edges with it belong to the other system of functions. Thus, in a circuit through the top of Fig. 15, the four small tetrahedra alternate from one to the other system, e.g., conscious to unconscious and so forth. The same property holds for the bottom



(a) (b)

Figure 18: The planar (a) Ho T'u diagram is represented with (b) a merkaba using a blue heavenly (odd numbers) tetrahedron and a red earthly (even numbers) tetrahedron. The green lines emphasize where the heavenly numbers lie between the earthly numbers, and the lines in (b) correspond to the arrows in (a).

of Fig. 15, but offset by one step, i.e., the unconscious is at the bottom when consciousness is at the top and vice versa.

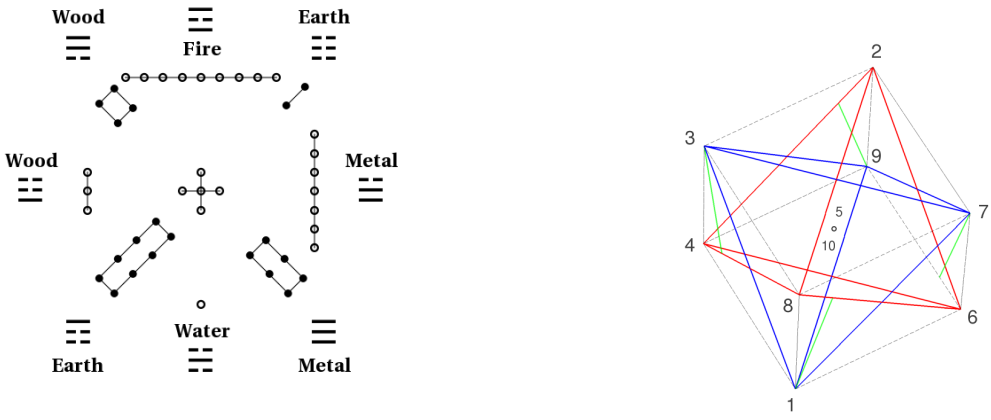
Applying these observations to Fig. 17a shows that a circuit through the creatures alternates from one to the other system, i.e., consciousness to unconscious and so forth. The same holds in a circuit through the wheel systems, but offset by one step, i.e., a wheel system at the bottom belongs to the unconscious system when the creature just above it at the top belongs to the conscious system, and vice versa.

To view Jung's algebraic formula in Fig. 16b as a top-down perspective of the merkaba in Fig. 17a suggests that each of Jung's quaternia should correspond to a pair of small tetrahedra, the creature above with its wheel system just below. Such a pair has either the creature in consciousness and the wheel system in the unconscious or vice versa. If the corners of a given quaternion are mapped to the tetrahedral faces both for the creature and for the wheel system, the quaternion is partly in consciousness and partly in the unconscious, one part mirrored by the other across the threshold to consciousness. This assumption is consistent with Jung's idea that consciousness is brought forth from the unconscious as one goes through the phases of his model, i.e., all states possess a mixture of consciousness and unconsciousness.

With the quaternia a_i, b_i, c_i, d_i in Fig. 16b identified with tetrahedral faces as just described, A, B, C, D in the cycle of Fig. 16b are identified each with a vertical edge pair connecting north and south poles of the octahedron in Fig. 17c. While preserving the shape of the merkaba, the octahedron may be rotated in 90° steps along the bottom-to-top axis, either clockwise or counterclockwise. Similarly, the small tetrahedra may be rotated independently in 120° steps along the axis from merkaba center to tetrahedral vertex, either clockwise or counterclockwise. Jung intended such motions to correspond to stages of individuation, which can run progressively or regressively, i.e., clockwise or counterclockwise.

4.4 Yellow River Maps and their Trigram Arrangements

For the forthcoming constructions for the I Ching in the next section, I considered to integrate the famous Chinese symbols shown in Figs. 18a and 19a, especially because of the attention given to these symbols in [36]. The dot sequences, • and ○, form the so-called Yellow River Maps, i.e., Ho T'u in Fig. 18a and Lo Shu in Fig. 19a. The trigram sequences around the two Yellow River Maps are called the Earlier and Later Heaven Arrangements, respectively [27]. The elements, metal, water, wood, fire and earth, are



(a) (b)

Figure 19: The planar (a) Lo Shu diagram is represented with (b) a merkaba using a blue heavenly (odd numbers) tetrahedron and a red earthly (even numbers) tetrahedron. The green lines emphasize where the heavenly numbers lie between the earthly numbers.

distributed around each map boundary, and they correspond to the Wuxing modes of transformation [38]. Further details about these celebrated symbols are found in [27], [36] and [38].

It was not possible to integrate the symbols of Figs. 18 and 19 with the constructions of forthcoming sections. Nevertheless, the effort bore fruit as I delved into these symbols. The work led to the merkaba representations shown in Figs. 18b and 19b. In these representations, the Chinese dot numerals (numbers of dots • or ○) at the left are mapped to the western Arabic numerals at the right, and these mappings carry the trigrams and elements to corresponding locations between the numbers. For me, the center carries the same meaningful correspondence to the Self here as with the other geometric symbols presented.

5 Edge Consistent Arrangement of Hexagrams

5.1 The Basic I Ching Table

The tetrahedral representation of hexagram 63 is demonstrated in Figs. 8 and 9. Continuing in this way gives the tetrahedral representation for all the 64 hexagrams of the I Ching [36]. The result is shown below in Fig. 20 in my own edge-consistent format. This format borrows heavily from the arrangement by S.H. Cullinane [2], which was developed for the box representation of hexagrams developed by M.-L. von Franz, as indicated earlier. In Fig. 20, every square form is a projected tetrahedron like that shown in Fig. 9b for hexagram 63. The arrangement is chosen so that the outer edges of every ⊠-symbol match those of the immediate neighbors.

Furthermore, for symbols on the boundary of the table, the consistency holds over the edge of the table after wrapping left around to right or bottom around to top. This property means that the table of Fig. 20 satisfies toroidal boundary conditions, i.e., the outermost edges match at the left and right boundaries and at the top and bottom boundaries. When such a flat square is rolled up to join left and right boundaries, it forms a seamless cylinder; then, when the two ends of the cylinder are joined, there results a seamless torus [34], with the form seen in Fig. 21a. Carrying out such a construction for the entries of the table in Fig. 20 leads to the tiling shown in Fig. 21b.

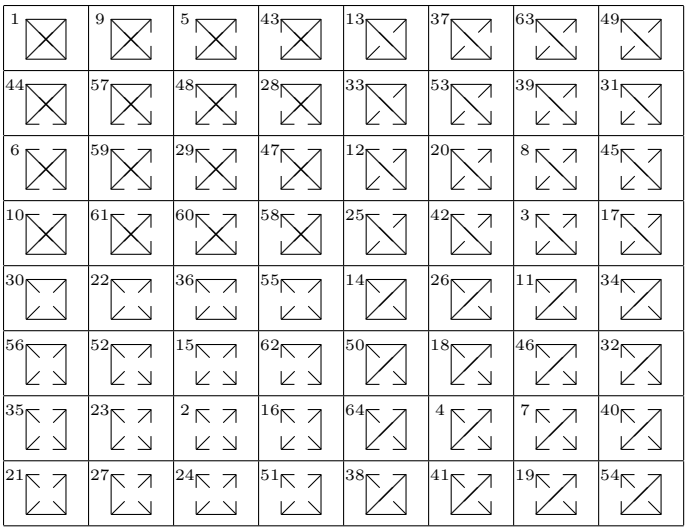
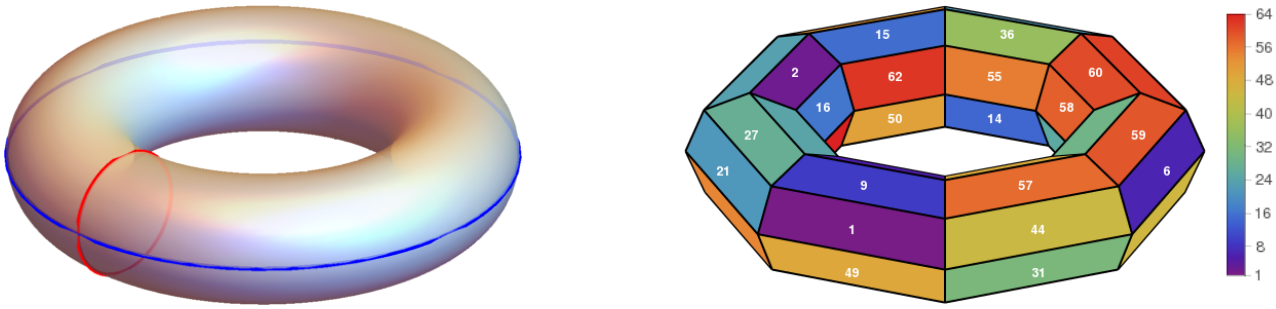


Figure 20: An I Ching table of hexagrams shown in their tetrahedral representations according to Fig. 9. The ordering satisfies edge-consistency, i.e., the outer edges of every ⊠-symbol match those of the immediate neighbors.



(a) (b)

Figure 21: (a) A torus has the shape of a tire tube or of a doughnut, with circular cross-sections in both polar (blue) and azimuthal (red) directions. (b) A toroidal representation of the table in Fig. 20.

5.2 The Complete Set of I Ching Outcomes

Discovering a toroidal rendering of edge-consistent tetrahedral representations of I Ching hexagrams is satisfying, but the construction represents only the 64 hexagrams. So I then considered whether there also exists a toroidal representation of the 64^2 possible outcomes of consulting the I Ching. I found such a representation, which is illustrated in Fig. 22.

The numbers in the table of Fig. 22 are the same numbers 1 – 64 used in the I Ching for its 64 hexagrams; see [36]. The top row contains the 64 possible present hexagrams. For any element in this row, the elements in the corresponding column represent the potential future hexagrams after changing lines are changed. Thus, the respective axes represent present and future times scales.

The table of Fig. 22 consists of 8×8 blocks of 64 numbers each, where each block is partitioned from its neighboring blocks by the horizontal and vertical lines in the table. The top left block is the same as the 8×8 table of Fig. 20. Each row of blocks is obtained in steps to the right by shifting the first row of numbers to be the last. These shifts are highlighted by marking the first row of the top left block in red. Each column of blocks is obtained in steps downward by shifting the first column of numbers to be the last. These shifts are highlighted by marking the first column in the top left block in blue. The number

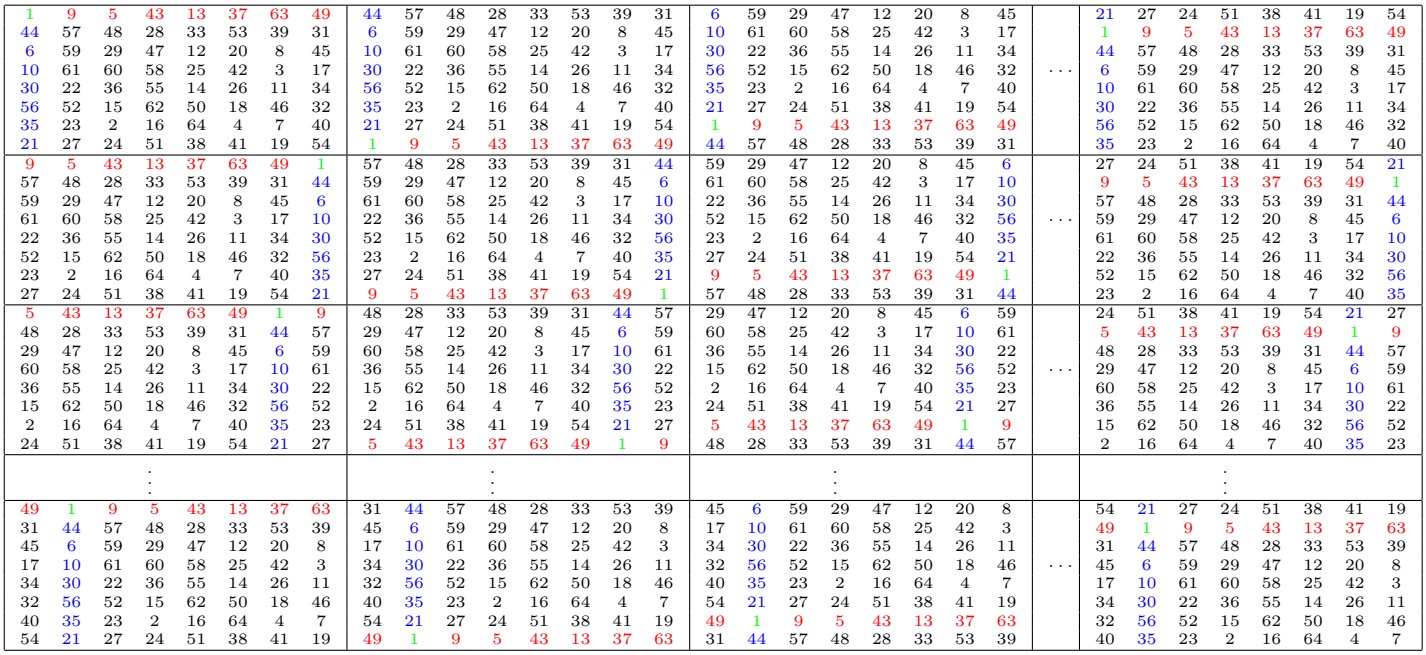


Figure 22: A table representing the total $64 \times 64 = 4096$ of all possible outcomes of consulting the I Ching. The top row contains the 64 possible present hexagrams. For any element in this row, the elements in the corresponding column represent the potential future hexagrams after changing lines are changed. The respective axes represent present and future times scales. The tetrahedral representations of the hexagrams satisfy neighbor consistency and are used to form a toroidal rendering of the table.

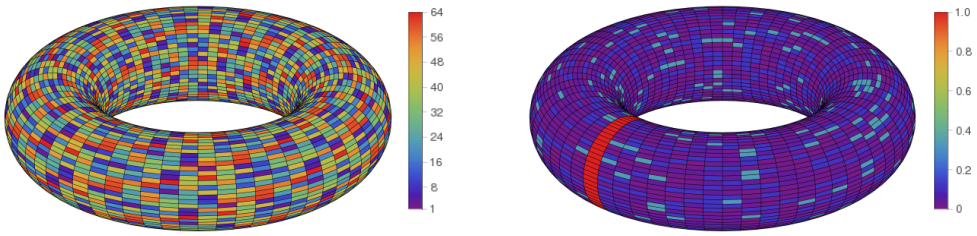
1 is in both color categories, so it is marked with the color green.

The numbers in the table of Fig. 22 are not only the same numbers of the I Ching, but they correspond also to the tetrahedral representation of hexagrams as shown in Fig. 20. All these tetrahedra satisfy edge consistency with their neighbors, which can be confirmed by checking every single connection. Yet there is a more convenient way of confirming the consistency, which is explained as follows.

First, recall that the table of Fig. 20 can be mapped onto a torus, as seen in Fig. 21b. By rotating this torus stepwise along its polar and/or azimuthal axes, one becomes none other than all the blocks shown in the table of Fig. 22. Thus, every block necessarily satisfies edge consistency within itself.

To verify edge consistency among blocks, note that between every two adjacent rows or between every two adjacent columns in the table of Fig. 20, the edges are either all yin or all yang. So at every block boundary in the table of Fig. 22, the edges are either all yin or all yang! For instance, in the first two rows of blocks, all vertical boundaries have only yang edges; in the third row of blocks, all vertical boundaries have only yin edges; and so forth. Similarly, in the first two columns of blocks, all horizontal boundaries have only yang edges; in the third column of blocks, all horizontal boundaries have only yin edges; and so forth. Thus, the entire table of tetrahedra satisfies edge consistency among neighbors.

The table of tetrahedra in Fig. 22 satisfies toroidal boundary conditions, just like those of Fig. 20. To represent this geometrically, the hexagram numbers of the table are mapped onto a torus as shown in Fig. 23a. The probabilities of these outcomes may be computed based upon the probabilities of Fig. 2. These outcome probabilities are shown mapped onto a torus in Fig. 23b. The highest probabilities are shown in the red ring of Fig. 23b, which corresponds to the first row of Fig. 22. This means that the most probable outcomes are those with no changing lines.



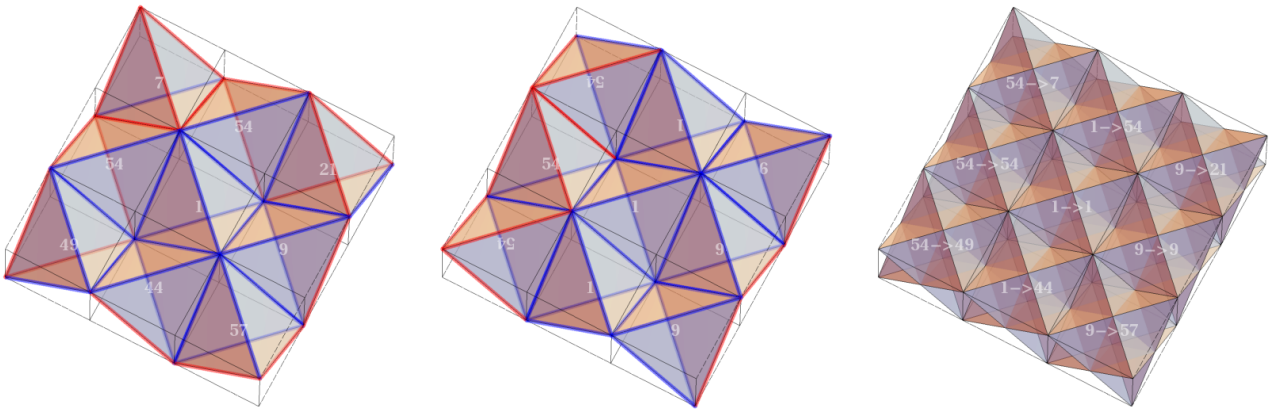
(a) (b)

Figure 23: (a) Hexagram numbers of the table in Fig. 22 are mapped onto the torus. (b) Probabilities of these outcomes, based on the probabilities of Fig. 2, are similarly mapped onto the torus.

5.3 The Merkaba Tiling of Outcomes

For a thoroughly explicit representation of the possible outcomes of consulting the I Ching, the pairing between a present and a future hexagram is represented by a pairing between the corresponding present and future tetrahedra into a single merkaba. This means that the top row of tetrahedra in Fig. 22 is paired with every other row to form 64^2 merkabas. Note that the first row is also paired with itself, since it is possible that no lines are changing.

Neighboring tetrahedra in the table of Fig. 22 are illustrated in Fig. 24a. The hexagram numbers are marked in white, and the yin- and yang-edges are marked in red and blue, respectively, instead of the with broken or solid lines as usual. With focus on a 3×3 part of the upper left corner of the table, and wrapping over table edges, the array of hexagram numbers is: top row $\{7, 54, 21\}$, middle row $\{54, 1, 9\}$ and bottom row $\{49, 44, 57\}$. Corresponding tetrahedra from the first row of the table of Fig. 22 are illustrated in Fig. 24b. These have hexagram numbers $\{54, 1, 9\}$ in all three rows. The resulting merkabas are illustrated in Fig. 24c with hexagram number pairs: top row $\{54 \to 7, 1 \to 54, 9 \to 21\}$, middle row $\{54 \to 54, 1 \to 1, 9 \to 9\}$ and bottom row $\{54 \to 49, 1 \to 44, 9 \to 57\}$. Once this construction is completed for the entire table of Fig. 22, there results a flat tiling of the I Ching outcomes. Since toroidal boundary conditions are satisfied, the merkabas are rolled up to join left and right boundaries to create a seamless tiled cylinder; then, the two ends of that cylinder are joined to create a seamless tiled torus.



(a) (b) (c)

Figure 24: For visual clarity, yin and yang edges are respectively marked red and blue, instead of with broken or full lines, as usual. (a) Nine adjacent tetrahedra from the table of Fig. 22. The hexagram numbers are marked in white. (b) Corresponding adjacent tetrahedra from the first row of the table of Fig. 22. The repeating hexagram numbers are also here marked in white. (c) Combining these gives the corresponding merkabas for the outcomes of the I Ching.

6 Pauli's World Clock Vision

6.1 The Vision

The toroidal representation of the I Ching has a surprising connection with Pauli's World Clock Vision. This vision is recounted below as reported in [16], §307 along with related details. A known illustration of the World Clock (e.g., with the black bird) appears in [31] (p. 84), but for the present purposes it is illustrated in Fig. 25 during the course of pulse sequences. Pauli had this vision toward the end of his analysis, and he described it as an image of *the most sublime harmony*. M.-L. von Franz associated the vision with two time scales, linear and eternal [35]. A musician might understand these scales and the perceived harmony in terms of a polyryhthm [29].

Vision: There is a vertical and a horizontal circle, having a common center. This is the world clock. It is supported by the black bird. The vertical circle is a blue disc with a white border divided into $4 \times 8 = 32$ *partitions. A pointer rotates upon it. The horizontal circle consists of four colors. On it stand four little men with pendulums, and round about it is laid the ring that was once dark and is now golden (formerly carried by the children). The "clock" has three rhythms or pulses:*

1. *The small pulse: the pointer on the blue vertical disc advances by 1/32.*
2. *The middle pulse: one complete revolution of the pointer. At the same time the horizontal circle advances by 1/32.*
3. *The great pulse: 32 middle pulses are equal to one revolution of the golden ring.*

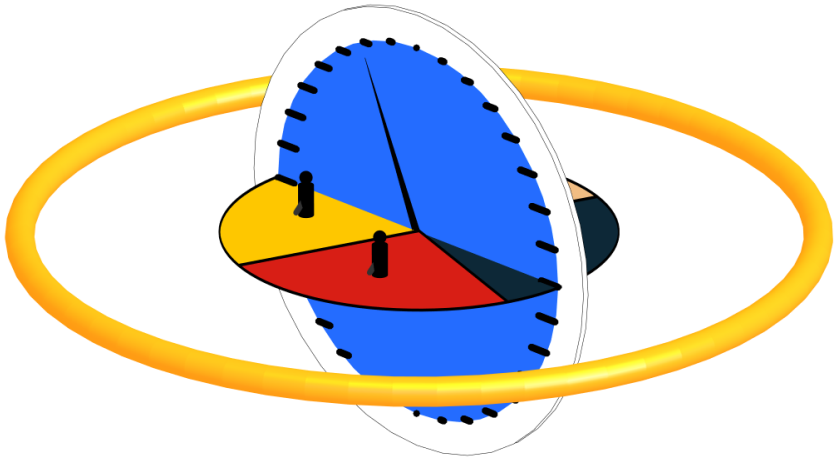


Figure 25: The World Clock envisioned by Wolfgang Pauli, which he described as an image of *the most sublime harmony*.

6.2 Connection to the I Ching

Consider the numbers appearing in this vision. For each middle pulse there are 32 small pulses. After 32 middle pulses, the golden ring revolves once. Thus, this revolution of the golden ring occurs after 32×32 small pulses. There are four pendulums, each associated naturally with its own color and alchemical stage. So four revolutions of the golden ring correspond to $4 \times 32 \times 32 = 64 \times 64$ small pulses.

Some colleagues from the C.G. Jung Institute Zurich may remember our discussion of the number of pulses in a full cycle of the World Clock. We assumed (incorrectly, perhaps because of the "three rhythms") that one great pulse corresponds to $32 \times 32 \times 32$ ($= 32^3$) small pulses. This assumption was strengthened after the numbers 2, 3 and 5 in $32^3 = (2^3)^5$ were observed to lie at the beginning of the Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots$ [4]. The special quality of Pauli's vision was then assumed to emerge from the convergence of the quotients of consecutive entries $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \ldots$ to the Golden Ratio,

$(1+\sqrt{5})/2$ [8]. I confess that the beauty of this picture distracted me from the initial error in the assumed count. A careful reading of Pauli's words shows, as explained above, that one revolution of the golden ring corresponds to 32^2 small pulses, not 32^3. Thus, 4 revolutions of the golden ring correspond to $4 \times 32^2 = 64^2$ small pulses.

Yet the special quality of the vision may indeed be revealed by the number 64×64. This is the number of small pulses in four revolutions of the golden ring as well as the number of possible results of consulting the I Ching. Thus, the merkaba-tiled torus constructed earlier for the I Ching can just as well be associated with the World Clock. All the possible outcomes of the I Ching and all the possible states of the World Clock may be distributed among the merkabas of the torus. For both the I Ching and for the World Clock, the two time scales, near- and far-term, extend over the two circular axes of the torus, polar and azimuthal. No wonder Pauli felt a sublime harmony with his image.

There is also a natural interpretation of each of the four revolutions of the golden ring, given as follows. Let the hexagrams of the I Ching, numbered 1 to 64, be partitioned into odd and even. Let the odd-numbered be designated as yang hexagrams and the even-numbered as yin hexagrams. As noted earlier, and explained in [36], yin and yang are associated with earth and heaven, respectively.

Using these designations, there are four natural cases of present to future hexagram pairs among the I Ching outcomes: earth to earth, earth to heaven, heaven to earth and heaven to heaven. These four cases may be associated, respectively, with the four quaternia in Jung's model of the Self: Lapis, Paradise, Shadow and Anthropos.

The number of outcomes in each of these four cases is the same, namely 32×32. Thus, one revolution of the golden ring in the World Clock corresponds to a complete transit through all I Ching outcomes in one of these four cases. Four revolutions of the golden ring correspond to a complete transit through all possible I Ching outcomes. In alignment with the quaternia of Jung's model, four revolutions of the golden ring may thus be associated with an alchemical realization of the Self.

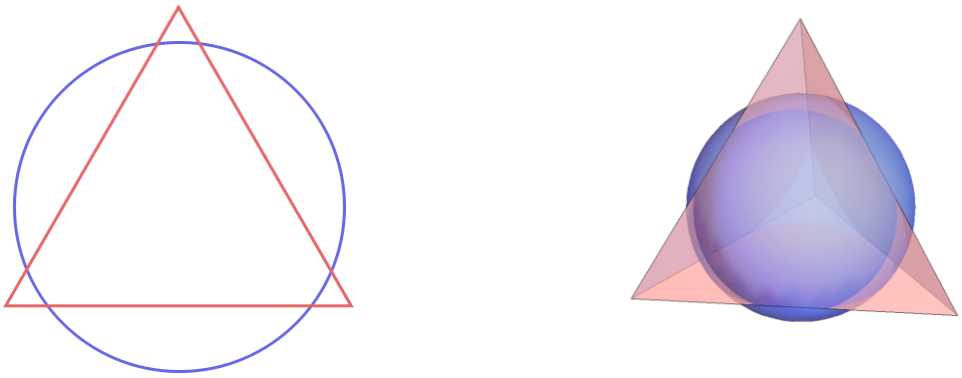
7 Symbols for Creativity, for Freedom

The symbol shown in Fig. 26a emerged in [20] to represent this entire work, in fact, to honor and to thank Kurt Gödel, as his accomplishment reveals the existence of true creativity, of true freedom. The symbol was sketched spontaneously by my wife, Brigitte Koris-Keeling, who knew only roughly about the topic of the article. As pointed out in [20], it is remarkable how the symbol in Fig. 26a resembles the Borromean rings of Christianity [1] and the Trimurti of Hinduism [**?**]. Further amplifications of the symbol follow.

The symbol in Fig. 26a may be associated naturally with the Philosopher's Stone because the construction of the symbol is very subtle; see also [22]. Specifically, the unit circle and the triangle shown have an identical perimeter, namely 2π. Because of the properties of π, which were proved as recently as in 1882 [21], it is not possible to "triangle the circle". This means it is not possible to construct a triangle with the same perimeter as a given circle in a finite number of steps using only simple geometric tools. Thus, it is natural to associate the alchemical work with this difficult task.

The simple creation in Fig. 26a corresponds to a task transcending finite means, and it is meant to symbolize Gödel's creation which corresponds to a truth transcending the Ego's means. As demonstrated in [20], that truth is the existence of genuine creativity in general and of genuine freedom in particular.

As with Pauli's Hexagram Dream, there is a natural and meaningful 3D correspondence to Fig. 26a, which is shown in Fig. 26b. This 3D figure shows a tetrahedron and a sphere, the two having one common center and the same surface area, namely 4π. I associate the vertices of the tetrahedron with Body, Mind, Soul and Spirit, as these are clarified in the first footnote of [10]. I associate the shared



(a) (b)

Figure 26: (a) The unit circle and the triangle share the same center and have the same perimeter, namely 2π. (b) The unit sphere and the tetrahedron share the same center and have the same surface area, namely 4π.

center with the Self. In the 3D form of Fig. 26b, these five components are explicitly displayed, but I see them implicitly present also in the 2D form of Fig. 26a. That neither the round nor the faceted is entirely contained, one within the other, shows me a reciprocal bond between creator and created.

To close this supplement, I want to say openly what it is really that I see symbolized in Fig. 26b. I see the Body according to the Pauli-Jung Conjecture [1], parallel to the Psyche, so the psychic comprehension of the Body is metaphysical. Yet, according to my experience, the Body seems to provide boundaries for the Psyche, though the magnetic effect of drives may pull those boundaries to tightly [11] (p. 43). A balance between Body and Psyche seems to favor individuation.

I see the Soul as the deep well of potential *impressions* out of the unconscious, perhaps explicitly correlated with capacities of the right brain [23]. Following Kant [19], I distinguish the ineffable impression, as thing-in-itself, from its crystallization, as thing-in-representation, such as a dream image. Thus, I see the Soul as the ineffable and the Soul Image as the representation.

Similarly, I see the Mind as the deep well of potential *thoughts* out of consciousness, perhaps explicitly correlated with capacities of the left brain [23]. I distinguish the preconscious thought, as thing-in-itself, from its crystallization, as thing-in-representation, such as a mental handle [7]. Thus, I see the Mind as the ineffable and the Mind Image, or Persona, as the representation.

Following Jung [10], I see the Spirit as a superordinate form which sublimates the Soul and the Mind. My experience suggests that the Spirit plays a guiding role and corresponds to the Self as nearly as possible in human life. This work has made Jung's Liverpool Dream [12] ever more palpable for me, suggesting, as he said, that in search of the Self, one can go no further than to the center, and yet that center mysteriously embraces everything.

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